A Novel Expanded 16-Dimensional Constant Envelope Q^2PSK Constellation

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***Abstract*—We introduce a 16-dimensional constant-amplitude constellation that is generated by concatenating either four constant envelope quadrature-quadrature phase shift keying (CEQ^{2}PSK) symbols from Saha and Birdsall or four CEQ^{2}PSK symbols recently discovered by Cartwright and also introduced here. Our new constellation doubles the number of points available for data transmission without decreasing the distance between points or increasing energy, and may therefore be used in a trellis coded modulation (TCM) system without constellation expansion penalty. Because the new constellation has constant envelope, the modulation scheme becomes very attractive for nonlinear channels such as the magnetic recording channel or the satellite channel with traveling wave tube amplifiers.**

***Keywords*—Multidimensional constellation, constant envelope, quadrature-quadrature phase shift keying, constellation expansion, trellis coded modulation.**

I. INTRODUCTION

Considerable research effort has been devoted to developing modulation schemes that can overcome the challenges of bandwidth limited channels. Saha and Birdsall [1], [2] suggested an efficient use of available dimensions to improve the spectral efficiency of a communications system. They presented quadrature-quadrature phase shift keying (Q^{2}PSK) and constant envelope Q^{2}PSK (CEQ^{2}PSK). Q^{2}PSK is a 4-dimensional (4-D) scheme that uses two quadrature carriers and two data shaping pulses.

Constant envelope is desirable in nonlinear channels; it avoids the variations in phase produced by changing amplitude, which in turn has detrimental effects in the performance of coherent demodulators. CEQ^{2}PSK achieves constant envelope at the expense of bandwidth efficiency because the information rate is 3/(2T) for CEQ^{2}PSK while it is 2/T for non-constant Q^{2}PSK. Fortunately, however, CEQ^{2}PSK also provides a gain of 1.44 dB over non-constant Q^{2}PSK as shown in [5], which corrects the more optimistic value of 1.76 dB given in [1].

In this paper, we present a new set of eight 4-D points which is also a valid CEQ^{2}PSK constellation. Furthermore, we also introduce a new 16-D constellation of 8192 points with constant envelope. Our 16-D constellation is created by the union of the set produced by transmitting four of the original CEQ^{2}PSK or four of our new CEQ^{2}PSK points over four consecutive time intervals.

The rest of this paper is organized as follows: we present a brief review of Q^{2}PSK and CEQ^{2}PSK in Sections II and III, respectively. In section IV, we introduce the new CEQ^{2}PSK constellation discovered by Cartwright and our new expanded 16-D constellation. In section V, we briefly discuss a couple of nonlinear channels in which our modulation system could be used. Suggestions for further work are given in Section VI. Concluding remarks are given in Section VII with the main references following.

II. REVIEW OF Q^{2}PSK

Quadrature-quadrature phase shift keying (Q^{2}PSK) [1], [2] is a spectrally efficient modulation scheme that uses available signal space dimensions in a more efficient way than two dimensional schemes such as quadrature phase shift keying (QPSK) and minimum shift keying (MSK). Saha and Birdsall's scheme uses four available dimensions created by two data shaping pulses and two quadrature carriers. The Q^{2}PSK modulating signal set $\{s_i(t)\}$, $i = 1, \ldots, 4$, is made up of the following four orthogonal waveforms:

$$s_1(t) = \cos(\pi t/2T)\cos(2\pi f_c t),\ |t| \le T \quad (1a)$$

$$s_2(t) = \sin(\pi t/2T)\cos(2\pi f_c t),\ |t| \le T \quad (1b)$$

$$s_3(t) = \cos(\pi t/2T)\sin(2\pi f_c t),\ |t| \le T \quad (1c)$$

$$s_4(t) = \sin(\pi t/2T)\sin(2\pi f_c t),\ |t| \le T. \quad (1d)$$

The carrier frequency, f_c, should be n/4T where n ≥ 2, and T is the time duration of 2 bits. The original binary data stream is demultiplexed into four signals $\{a_i(t)\}$, $i = 1, ..., 4$, each of duration 2T. Each a_i(t) is then multiplied by the modulating signal s_i(t), and the resulting signals are added to form the modulated non-constant envelope signal S_q(t) [1]:

$$S_q(t) = \sum_{i=1}^{4} a_i(t) s_i(t)\,. \tag{2}$$

The number of possible symbols in this modulation technique is $2^4 = 16$ because there are four bits input to the modulator. The bit rate at the input of the modulator is 2/T which is twice the bit rate of the QPSK scheme.

III. Review of Constant Envelope Q^{2}PSK

Constant envelope, desired for non-linear channels, can be introduced to produce CEQ^{2}PSK, as presented in [1]. The modulated Q^{2}PSK signal of (2) may be rewritten as:

$$S_q(t) = A(t)\cos\left(2\pi f_c t + \theta(t)\right), \tag{3}$$

where $\theta(t)$ can be any of the four possible values: ±45°, ±135°. This phase shift produces discontinuities in phase and abrupt ±90° or ±180° phase changes in the Q^{2}PSK signal may occur at a symbol transition [2].

$A(t)$ in (3) is the carrier amplitude given in [1] as:

$$A(t) = \left(2 + \left(a_1 a_2 + a_3 a_4\right)\sin\frac{\pi t}{T}\right)^{1/2}, \tag{4}$$

where $\{a_i\}$, $i = 1, ..., 4$, are, respectively, the binary values of the input signals $\{a_i(t)\}$, $i = 1, ..., 4$, at time t, (n-1)T ≤ t ≤ nT (i.e., ±1). In order to accomplish constant envelope, the amplitude $A(t)$ in (4) must fulfill the following condition [1], [2]:

$$a_1 a_2 + a_3 a_4 = 0. \tag{5}$$

For CEQ^{2}PSK modulation we have three information input bits $\{a_i\}$, $i = 1, 2, 3$, while the fourth bit is produced by a simple block encoder of rate 3/4 where $a_4 = -a_1 a_2/a_3$ to satisfy (5). The eight possible symbols that Saha and Birdsall found are shown in Table I, and labeled C_i, $i = 1, 2, ..., 8$.

The set of points $\{C_i\}$, $i = 1, 2, ..., 8$, has peak energy of 2 per 2-D (or 4 over 4-bit interval), and the minimum squared Euclidian distance (MSED) between any pair of signal points is equal to 8.

In Table II, we show the distribution of squared distances between points in Saha and Birdsall's CEQ^{2}PSK constellation. Obviously, the eight points at distance zero are between a point and itself.

Table I
Saha and Birdsall's CEQ^{2}PSK symbols

		a_1	a_2	a_3	a_4
symbols	C_1	1	1	1	-1
	C_2	1	1	-1	1
	C_3	1	-1	1	1
	C_4	1	-1	-1	-1
	C_5	-1	-1	-1	1
	C_6	-1	-1	1	-1
	C_7	-1	1	-1	-1
	C_8	-1	1	1	1

Table II
Distance distribution of CEQ^{2}PSK

Squared Euclidian distance	Number of points
0	8
8	48
16	8

IV. The New CEQ^{2}PSK Constellations

The main contributions of this paper are contained in this Section. We present first a second set of valid 4-D CEQ^{2}PSK points in subsection A. In subsection B, we show the two 16-D constant envelope constellations, and in subsection C we introduce the expanded 16-D constellation which contains 8192 points, twice as many as needed to transmit 3 information bits per 4-D.

A. Cartwright's CEQ^{2}PSK constellation

We have found a new set of 8 symbols that is also valid for CEQ^{2}PSK. The new set $\{K_i\}$, $i = 1, 2, ..., 8$, has the same energy as the set $\{C_i\}$, $i = 1, 2, ..., 8$, and the same distribution of squared distances. Therefore, our constellation also has an MSED of 8.

Table III shows the novel eight symbols. Clearly, the constant envelope condition of (5) is satisfied by this new set, and the symbol energy is the same as that of the original set shown in Table I.

However, there is another constraint that a CEQ^{2}PSK constellation must satisfy that is not mentioned by Saha and Birdsall [1]: namely,

$$a_1^2 + a_3^2 = a_2^2 + a_4^2. \tag{6}$$

The validity of (5) and (6) is established by substituting (1) into (2) to get

$$\begin{aligned} S_q(t) = &\left(a_1 \cos\left(\frac{\pi t}{2T}\right) + a_2 \sin\left(\frac{\pi t}{2T}\right)\right)\cos 2\pi f_c t + \\ &\left(a_3 \cos\left(\frac{\pi t}{2T}\right) + a_4 \sin\left(\frac{\pi t}{2T}\right)\right)\sin 2\pi f_c t. \end{aligned} \tag{7}$$

Clearly, the amplitude of (7) is given by

$$A(t)=\begin{bmatrix}\left(a_1\cos\left(\frac{\pi t}{2T}\right)+a_2\sin\left(\frac{\pi t}{2T}\right)\right)^2+\\ \left(a_3\cos\left(\frac{\pi t}{2T}\right)+a_4\sin\left(\frac{\pi t}{2T}\right)\right)^2\end{bmatrix}^{0.5}. \quad (8)$$

Simplifying (8) gives

$$A(t)=\begin{bmatrix}\frac{a_1^2+a_2^2+a_3^2+a_4^2}{2}+\\ \left(\frac{a_1^2+a_3^2}{2}-\frac{a_2^2+a_4^2}{2}\right)\cos\left(\frac{\pi t}{T}\right)+\\ \left(a_1a_2+a_3a_4\right)\sin\left(\frac{\pi t}{T}\right)\end{bmatrix}^{0.5}. \quad (9)$$

It can be easily seen that (9) has constant envelope if (5) and (6) are satisfied, as they are for the constellation of Saha and Birdsall [1] and the new one introduced here in Table III. (By the way, (9) reduces to (4) for both constellations.)

B. The 16-D CEQ^{2}PSK Constellations

In [3], a constellation of 2n-dimensional (2*n*-D) points was proposed by transmitting *n* consecutive 2D in-phase and quadrature-phase pairs. A similar approach is followed here, where in order to obtain the expanded 16-dimensional constant envelope Q^{2}PSK constellation we use 4 consecutive 4-D CEQ^{2}PSK symbols, instead of using 8 consecutive 2D QPSK symbols.

If four consecutive 4-D symbol time slots are taken at once, a 16-dimensional symbol can be generated. We have two subsets with symbols of the form $S_{16a} = [C_i\ C_j\ C_p\ C_q]$, and $S_{16b} = [K_i\ K_j\ K_q\ K_p]$, where i, j, p, q = 1, 2, ⋯, 8. For the standard CEQ^{2}PSK over 4 consecutive time intervals, there are 8^4 = 4096 possible points $\{S_{16ai}\}$, *i = 1, 2, ..., 4096*. The set $\{S_{16bi}\}$, *i = 1, 2, ..., 4096*, contains the 4096 points formed by four consecutive points of Cartwright's CEQ^{2}PSK of Table III.

Table IV shows the squared distance distribution for either of these two sets. Because each 16-dimensional set has 4096 signals, the distribution has a total of $(4096)^2$ distances.

Table III
Cartwright's CEQ^{2}PSK symbols

		a_1	a_2	a_3	a_4
symbols	K_1	0	√2	√2	0
	K_2	0	-√2	-√2	0
	K_3	0	-√2	√2	0
	K_4	0	√2	-√2	0
	K_5	√2	0	0	√2
	K_6	√2	0	0	-√2
	K_7	-√2	0	0	√2
	K_8	-√2	0	0	-√2

Within the set $\{S_{16ai}\}$ or $\{S_{16bi}\}$, *i = 1, 2, ..., 4096,* the MSED between any pair of different points is still 8, and the peak energy is still equal to 2 per 2-D.

C. The New Expanded 16-D CEQ^{2}PSK Constellation

Our novel 16-dimensional constellation is the union of the set $\{S_{16ai}\}$ and $\{S_{16bi}\}$, yielding $2{\cdot}8^4$ = 8192 possible points. Notice this is not equivalent to the 4-fold Cartesian product constellation of $\{C_i\}\cup\{K_i\}$ because the four consecutive 4D symbols used to produce the 16-D symbols must come only from one or the other 4-D constellation. This affects the partitioning that will be needed for trellis coded modulation (TCM) using our expanded constellation.

The MSED between points of this new expanded constellation is still 8, which is the intra-set MSED of each of the two 16-D sets. The MSED across sets $\{S_{16ai}\}$ and $\{S_{16bi}\}$ is equal to $8(1+(\sqrt{2}-1)^2)$ = 9.373. This new constellation, then, has twice as many points within the same 16-D space, with the same energy per point, without decreasing the MSED. This will allow a TCM system to be developed that uses the expanded constellation without paying the usual constellation expansion penalty.

Table V shows the squared distances from points in $\{S_{16ai}\}$ to points in $\{S_{16bi}\}$, and the multiplicity of these. The complete squared-distance distribution for our expanded constellation is the union of those listed in Tables IV and V, with twice the number of pairs indicated; for example there are 196608 points at MSED 8 and 131072 pairs of points with SED 9.373.

Table IV
Distance distribution of the 16-D CEQ^{2}PSK constellations $\{S_{16ai}\}$ or $\{S_{16bi}\}$

Squared Euclidian distance	Number of pairs
0	4096
8	98304
16	901120
24	3833856
32	7102464
40	3833856
48	901120
56	98304
64	4096

Table V
Distance distribution between points in $\{S_{16ai}\}$ and points in $\{S_{16bi}\}$

Squared Euclidian distance	Number of pairs
9.373	65536
15.029	524288
20.686	1835008
26.343	3670016
32.00	4587520
37.656	3670016
43.313	1835008
48.970	524288
54.627	65536

V. NONLINEAR CHANNELS

Because nonlinear channels require constant envelope signals, our new constellation is a good option in channels such as the recording magnetic channel, and the travelling wave tube (TWT) channel. For this reason, we discuss very briefly these two channels.

A. Recording Channel

The digital magnetic recording channel is nonlinear due to a process called saturation magnetic recording [6]. This particular phenomenon has been modeled by using a Volterra series expansion.

Sands and Cioffi [7] suggest a system transfer function by using Discrete Volterra Series (DVS). According to Sands and Cioffi, the channel can be modeled with most of the nonlinear distortion represented with third-order terms, which allows for a relatively compact channel description. Signals for the recording channels should be DC-free and have constant envelope.

B. Traveling Wave Tube (TWT) Satellite Channel

The model of nonlinear TWT amplifiers presented by Saleh in [8] may be used for the satellite communications channel if the satellite amplifiers are being driven near to the saturation point. Constant amplitude modulation schemes such as the one presented in this paper allows operation of the amplifier in that situation, avoiding loss of power.

VI. FUTURE WORK

Because our new expanded 16-dimensional has redundant symbols, we can use this constellation along with a convolutional encoder to produce a novel multidimensional trellis coded modulation (TCM) system similar to those in [3], [4], but using CEQ^{2}PSK instead of QPSK and therefore gaining an additional advantage due to the better utilization of the signal dimensions. In order to accomplish this task, an adequate set partition has to be implemented.

Simulations of the CEQ^{2}PSK TCM system with our expanded 16-D constellation are being conducted and shall be presented in a companion paper. The hardware detector of [5] must be modified to optimally decode the new sets. The performance of the 16-D CEQ^{2}PSK-TCM system over nonlinear channels such as those mentioned in Section V must be determined.

VII. CONCLUSIONS

We have proposed a second set of 4-D constant envelope quadrature-quadrature PSK (CEQ^{2}PSK) signals comparable to those of Saha and Birdsall. Furthermore, we have used both of these CEQ^{2}PSK sets to create a novel 16-D constellation of 8192 points. The 16-D constellation is the union of all points formed by 4 consecutive 4-D points from one or the other CEQ^{2}PSK constellation. Our expanded 16-D constant envelope constellation allows redundancy to be introduced through a convolutional encoder in a TCM scheme without suffering any power penalty due to constellation expansion, as the MSED and the average and peak energies of the expanded constellation are the same as those of the system before expansion: the number of symbols is doubled without decreasing MSED or increasing power. Because this constellation has constant envelope, it is attractive for use in nonlinear channels. Optimal yet simple hardware detection is possible.

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