Session 10F2

Performance Of The Adaptive Baseband 4D Diagonalizer, A Crosstalk Canceller, With Four-Dimensional Constellations

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Abstract

A general formula for the probability of symbol error P_{es} performance for the diagonalizer with a four dimensional (4D) detector is described. This 4D diagonalizer is important for systems with depolarization crosstalk, which use 4D constellations rather than constellations which can be detected with independent 1D detectors such as dual channel M-QAM (DCM-QAM). The formula is applied to a 256 symbol 4D constellation (herein called GL4-256), which achieves 1.2 dB gain over 16-QAM for zero-crosstalk. It is shown that GL4-256 achieves better P_{es} performance than DC16-QAM for crosstalk levels below -9.5 dB, with the converse being true for higher crosstalk levels.

A block diagram of the adaptive baseband 4D diagonalizer is also provided.

Introduction

Digital communication systems can have increased route capacity by utilizing dual polarization of two transmitted information-bearing signals on the same carrier frequency. These systems will then be susceptible to crosstalk interference which causes degradation in system performance. This degradation is severe in practical systems, and steps must be taken to reduce this problem. Presently, there are three known cancellers which reduce the interference problem. These are: (i) the diagonalizer [1-3], (ii) the minimum mean square error (MMSE) canceller [2,4,5], and (iii) the maximum likelihood (MLD) receiver [4,6]. Duvoisin, et al. [3,5] obtained exact values for the probability of symbol error P_{es} in the dual-channel 16-QAM case, which showed the equivalence of the diagonalizer and the MMSE receiver for signal-to-noise ratios of normal interest. Cartwright, et al. [6] also obtained P_{es} performance for the MLD receiver using a truncated union upper bound for the dual-channel 16-QAM case, but the method presented there is also valid for any compact constellation which is a subset of a lattice. (By a compact constellation, we mean a constellation which retains all lattice points which are at a lower energy level than a maximum energy constellation point, so that the constellation retains the stacking property of the lattice.)

Gersho and Lawrence [7] have presented multi-dimensional constellations that are subsets of lattices, and they showed that their 4D-256 symbol constellation (hereafter called the GL4-256 constellation), which is a subset of a lattice which they call A_4, can achieve a 1.2 dB gain over 16-QAM. Gersho and Lawrence assumed that the 4D symbol can be formed by using two 2D symbols over two time slots. However, the dual-polarization channel allows the transmission of a 4D symbol over a single time slot, thereby allowing twice the information rate. However, if this is done, the effects of the depolarization crosstalk must be evaluated. The purpose of this paper is to compute the P_{es} performance of the diagonalizer using the GL4-256 constellation and to compare it to dual-channel 16-QAM.

Dual-Polarization System Model

In a dual-polarization system, a transmitted symbol can be written as $D = [a_1,b_1,a_2,b_2]^T$ where a_1, b_1, a_2, and b_2 are data sequences. The a_i sequence modulates the in-phase carrier of channel i, and b_i modulates the quadrature phase of channel i, i = 1,2. The union of all possible transmitted symbols is called the transmitted constellation, and the allowed values of the components of D depend upon this. For dual-channel M-QAM, a_i, b_i, i = 1,2, take on values from the set {±1, ±3,..., $\pm M^{1/2} - 1$}. For GL4-256, the components of D are all even or all odd integers. In fact, any symbol D is a permutation of the seven basic points shown in Table I, e.g. $[0,2,0,0]^T$ or $[1,1,3,1]^T$. In addition, sign variations of D are also allowed, e.g. $[0,-2,0,0]^T$ or $[-1,1,1,3]^T$.

Table I. Basic Points for GL4-256 Constellation.

Basic Points	Energy	Number of Points	Number of Neighbors
1. 1111	4	16	23
2. 0002	4	8	22
3. 0022	8	24	22
4. 0222	12	32	19
5. 2222	16	16	15
6. 1113	12	64	15
7. 1133	20	96	8

Average Energy is 14.125.

The channel is assumed to be slowly varying and nondispersive. Thus, the model is applicable to the satellite path which takes on new significance for multi-level constellations, since it has been recently demonstrated by Feher [8] that 16-QAM systems are possible over the satellite path. Also, according to Amitay [1], the model is applicable to terrestrial radio systems in the 4 GHz band for fades which do not exceed 30 dB. In addition, the channel corrupts the transmission by introducing a fraction of one information stream into the other. Also, zero-mean AWGN is present with variance σ^2 in each component of both channels.

Thus, the received symbol is written as

$$Z = TD + N \quad (1)$$

where Z is the received symbol, i.e. $Z = [x_1, y_1, x_2, y_2]^T$, with x_i as the received sequence in the in-phase demodulator of channel i, and y_i as the received sequence in the quadrature demodulator of that same channel, i = 1,2.

N is the noise vector $[N_{1R}, N_{1I}, N_{2R}, N_{2I}]^T$. T is the channel matrix, given as

$$T = \begin{bmatrix} 1 & 0 & r\cos\theta_1 & -r\sin\theta_1 \\ 0 & 1 & r\sin\theta_1 & r\cos\theta_1 \\ r\cos\theta_2 & -r\sin\theta_2 & 1 & 0 \\ r\sin\theta_2 & r\cos\theta_2 & 0 & 1 \end{bmatrix}$$

with r as the crosstalk level, and the crosstalk phases θ_1 and θ_2 are distributed between $-\pi$ and π. This channel matrix T follows from the assumptions made by Nichols, et al. [4]. In the absence of crosstalk, r = 0, and T becomes the identity matrix.

In the absence of noise, the totality of possible received symbols is defined to be the received distorted constellation, which is easily shown to be a subset of another infinite lattice.

The conventional QAM detector operates on the received symbols Z to make four individual 1-D decisions to arrive at an estimate for D, whereas the GL4-256 detector makes one 4D decision. The diagonalizer first multiplies (1) by another matrix $W = T^{-1}$ to give the diagonalized symbol

$$Z_D = WZ = D + T^{-1}N. \qquad (2)$$

The symbol Z_D is then operated on by the conventional QAM detector, or the GL4-256 4D detector. Figure 1 shows the adaptive 4D diagonalizer, which is an extension of the one for DCM-QAM, as given by Kavehrad [2]. Complex notation is used for convenience only.

Probability of Symbol Error

(i) Zero-Crosstalk Case

When the crosstalk is zero, computing the probability of symbol error performance reduces to finding P_{es} for the constellation, which is relatively straightforward as discussed by Conway and Sloane [9].

Assume that symbol D_k has been transmitted. Then the decision region for D_k, i.e. the region in R^4 closer to D_k than to any other symbol, is determined by V_k symbols surrounding D_k. These symbols are called the relevant symbols D_r for D_k. For dual-channel M-QAM (DCM-QAM) and GL4-256, the relevant symbols are at a squared distance of 4 away from D_k, as shown by Gersho and Lawrence [7]. Thus, defining $S_{kr} = D_k - D_r$, then S_{kr} is any permutation or sign variation of $[0,0,0,2]^T$ for dual-channel M-QAM, and for the GL4-256, S_{kr} also includes any sign variation of $[1,1,1,1]^T$. Hence, $V_{max} = 8$ for DCM-QAM, and $V_{max} = 24$ for GL4-256. Thus, we can write

$$P_{es}(T|D_k) \le \sum_{r=1}^{V_k} \frac{1}{2}\,\mathrm{erfc}\left[\frac{\|S_{kr}\|^2}{8\sigma^2}\right]^{\frac{1}{2}} \qquad (3)$$

where $\|S_{kr}\|^2$ is the Euclidean norm. Clearly then, the probability of symbol error can be gotten by averaging (3) over the entire constellation.

$$P_{es}(T) \le \sum_{k=1}^{L} P_{es}(T|D_k)P(D_k) \qquad (4)$$

where L = total number of symbols in the constellation.

Usually, $P(D_k) = 1/L$ and so (4) becomes

$$P_{es}(T) \le \frac{1}{2L} \sum_{k=1}^{L} \sum_{r=1}^{V_k} \mathrm{erfc}\left[\frac{\|S_{kr}\|^2}{8\sigma^2}\right]^{\frac{1}{2}} \qquad (5)$$

A cursory glance at (5) indicates there are

$\sum_{k=1}^{L} V_k$ of the $\|S_{kr}\|$ distances to be computed.

However, for practical constellations, $\|S_{kr}\|$ takes on only a few values. This can be seen by writing S_{kr} as $[A,B,C,D]^T$, where $A = a_{1k} - a_{1r}$, $B = b_{1k} - b_{1r}$, $C = a_{2k} - a_{2r}$, and $D = b_{2k} - b_{2r}$. However, A, B, C, D are quantized since the components of the constellation symbols are quantized. Thus, $\|S_{kr}\|^2 = A^2 + B^2 + C^2 + D^2$ is also quantized. (Actually, $\|S_{kr}\|^2$ has only one value for GL4-256 and DCM-QAM. However, to maintain complete generality for arbitrary constellations, we will assume that $\|S_{kr}\|^2$ has N_L values.) Given this, (5) can be rewritten as

$$P_{es}(T) \le \frac{1}{2L} \sum_{I=1}^{N_L} N_{RI}\,\mathrm{erfc}(R_I/8\sigma^2)^{\frac{1}{2}} \qquad (6)$$

where N_{RI} is the number of squared distances having value R_I, and R_I is the I^{th} value of $\|S_{kr}\|^2$. N_L is the number of unique R_I values, hereafter called R_I levels. Both N_{RI} and N_L can easily be found by a computer algorithm.

For DCM-QAM, $N_L = 1$ and $N_{R1} = 8\left(\frac{M^{\frac{1}{2}} - 1}{M^{\frac{1}{2}}}\right)M^2$,

whereas for GL4-256, $N_L = 1$ and $N_{R1}/256 = 14.25$, which is the average number of relevant symbols [7].

(ii) Crosstalk Case

The calculation of probability of symbol error for the crosstalk case is similar to the zero-crosstalk case in that the decision boundaries are the same when the diagonalizer is used. Assuming that D_k has been transmitted, the decision boundary between D_k and a relevant symbol D_r is the hyperplane located midway between D_k and D_r, with equation given by

$$D^T S_{kr} - W = 0 \qquad (7)$$

where $W = \frac{1}{2}(D_k^T D_k - D_r^T D_r)$.

Now D_k is on the positive side of this boundary. Thus, the probability of symbol error given that D_k is transmitted is the probability that $Z_D^T S_{kr} - W < 0$, or

$$P_{scr}(T|D_k) = P(Z_D^T S_{kr} < W|D_k). \quad (8)$$

Fortunately, (8) is straightforward to evaluate since $Z_D^T S_{kr}$ is Gaussian with mean $D_k^T S_{kr}$ and variance $\sigma_D^2 = \sigma^2 S_{kr}^T (T^T T)^{-1} S_{kr}$. Therefore, (8) becomes

$$P_{scr}(T|D_k) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{D_k^T S_{kr} - W}{\sqrt{2}\,\sigma_D}\right) \quad (9)$$

But $D_k^T S_{kr} - W = \|S_{kr}\|^2/2$, and so (9) becomes

$$P_{scr}(T|D_k) = \frac{1}{2}\,\mathrm{erfc}\left[\frac{\|S_{kr}\|^4 (1+r^4 - 2r^2 \cos\theta)}{8\sigma^2 S_{kr}^T Q S_{kr}}\right]^{\frac{1}{2}} \quad (10)$$

where $Q = (1 + r^4 - 2r^2 \cos\theta)(T^T T)^{-1}$ and can be shown to be

$$Q = \begin{bmatrix} 1+r^2 & 0 & -rPC & rPS \\ 0 & 1+r^2 & -rPS & -rPC \\ -rPC & -rPS & 1+r^2 & 0 \\ rPS & -rPC & 0 & 1+r^2 \end{bmatrix}$$

with $PS = \sin\theta_1 - \sin\theta_2$, $PC = \cos\theta_1 + \cos\theta_2$, and $\theta = \theta_1 + \theta_2$.

It is also straightforward to show that

$$S_{kr}^T Q S_{kr} = (1+r^2)\|S_{kr}\|^2 - FrPC - GrPS \quad (11)$$

where

$$F = 2(AC+BD)$$

$$G = 2(BC-AD).$$

This is very similar to the distances used to calculate P_{es} for the MLD receiver [6]. However, in [6], (11) represents distances between symbols in the distorted constellation, whereas here (11) represents noise variance levels. Note that F and G are quantized, so that (11) takes on only a limited number of values. As a matter of fact, each unique $R_I = \|S_{kr}\|^2$ level gives rise to a limited number of values for A, B, C, and D, which in turn gives rise to a limited number of unique values of the (F,G) pair. Thus, the zero-crosstalk R_I level splits up into N_I sub-levels, where N_I is the number of unique (F,G) pairs, when crosstalk is added. Use will be made of this fact to reduce the computational burden.

Substituting (11) into (10) gives the probability of symbol error performance, given that D_k is transmitted, for one boundary only, that is, the boundary due to D_r. The total probability of symbol error given that D_k is transmitted is given by

$$P_{sc}(T|D_k) \le \sum_{r=1}^{V_k} P_{scr}(T|D_k) \quad (12)$$

The average probability of symbol error $P_{sc}(T)$, assuming each symbol is equally likely, is given by

$$P_{sc}(T) \le \frac{1}{L}\sum_{k=1}^{L}\sum_{r=1}^{V_k} P_{scr}(T|D_k)$$

$$\le \frac{1}{2L}\sum_{i=1}^{L}\sum_{r=1}^{V_k} \mathrm{erfc}\left[\frac{\|S_{kr}\|^4(1+r^4-2r^2\cos\theta)}{8\sigma^2[(1+r^2)\|S_{kr}\|^2 - FrPC - GrPS]}\right]^{\frac{1}{2}} \quad (13)$$

Note that, for zero crosstalk, (13) reduces to (5) as it should.

Again, a casual inspection of (13) indicates there are $\sum_{k=1}^{L} V_k$ complementary error functions to be calculated. However, as stated earlier, $\|S_{kr}\|^2$ is quantized to levels of R_I, there being N_L of these, and each R_I level of (11) splits up into N_I sub-levels, each with value $R_{IJ} = (1+r^2)R_I - F_{IJ}rPC - G_{IJ}rPS$, where F_{IJ}, G_{IJ} = F,G at the IJ^{th} sublevel. Therefore, (13) can be written as

$$P_{sc}(T) \le \frac{1}{2L}\sum_{I=1}^{N_L}\sum_{J=1}^{N_I} N_{IJ}\,\mathrm{erfc}\left[\frac{R_I^2(1+r^4-2r^2\cos\theta)}{8\sigma^2 R_{IJ}}\right]^{\frac{1}{2}} \quad (14)$$

where N_{IJ} is the number of sub-levels with value of R_{IJ}.

For zero-crosstalk, $R_{IJ} = R_I$, $N_I = 1$, $N_{IJ} = N_{RI}$, and (14) becomes (6) as it should. Note that $\sum_{J=1}^{N_I} N_{IJ} = N_{RI}$. N_{IJ}, R_{IJ}, N_I can be found with a slightly modified version of the computer algorithm discussed in [6] and given below:

<u>Step 1</u>. Select a symbol D_k in the transmitted constellation.

<u>Step 2</u>. Choose another symbol D_j ($j \ne k$). Compute $R_I = \|D_k - D_j\|^2$. If $R_I \ne 4$, go to Step 4 (this removes non-relevant symbols); otherwise, compute F and G.

<u>Step 3</u>. Each unique combination of F and G gives rise to a unique IJ value. Hence, increment the N_{IJ} counter for each unique combination.

Step 4. Repeat Steps 2 and 3 until all L symbols have been chosen in Step 2.

Step 5. Repeat Steps 1 to 4 until all L symbols have been chosen in Step 1.

For DCM-QAM, $N_L = 1$, $N_I = 1$, $N_{11} = N_{R1}$, and $R_I = 4$. Therefore, using (14), the probability of symbol error for DCM-QAM, $P_{sc1}(T)$, becomes

$$P_{sc1}(T) \le 4\left(\frac{M^{\frac{1}{2}}-1}{M^{\frac{1}{2}}}\right)\text{erfc}\left[\frac{1+r^4-2r^2 \cos\theta}{2\sigma^2(1+r^2)}\right]^{\frac{1}{2}} \qquad (15)$$

This agrees with the value found in [3] for DC16-QAM. For zero-crosstalk r = 0, and (15) reduces to the correct value. For GL4-256, $N_L = 1$, $N_I = 5$, and the other relevant values for variables to be used in (14) are shown below in Table II.

Table II. Values to Calculate $P_{es}(T)$ for GL4-256 Constellation.

Level I	Sublevel J	N_{IJ}	F_{IJ}	G_{IJ}
1	1	1376	0	0
	2	568	0	-4
	3	568	0	4
	4	568	-4	0
	5	568	4	0

Using Table II and (14), the probability of symbol error for GL4-256, $P_{sc2}(T)$, becomes

$$P_{sc2}(T) \le \sum_{i=1}^{5} P_i(T) \qquad (16)$$

where

$$P_1(T) = \frac{1376}{512}\ \text{erfc}\left[\frac{PD}{1+r^2}\right]^{\frac{1}{2}}$$

$$P_2(T) = \frac{568}{512}\ \text{erfc}\left[\frac{PD}{1+r^2-rPC}\right]^{\frac{1}{2}}$$

$$P_3(T) = \frac{568}{512}\ \text{erfc}\left[\frac{PD}{1+r^2+rPC}\right]^{\frac{1}{2}}$$

$$P_4(T) = \frac{568}{512}\ \text{erfc}\left[\frac{PD}{1+r^2-rPS}\right]^{\frac{1}{2}}$$

$$P_5(T) = \frac{568}{512}\ \text{erfc}\left[\frac{PD}{1+r^2+rPS}\right]^{\frac{1}{2}}$$

with $PD = (1 + r^4 - 2r^2 \cos\theta)/2\sigma^2$.

If it is assumed, as is usually done, that the crosstalk phases are uniformly distributed over $-\pi$ to π, then the average probability of symbol error is

$$\langle P_{esi}\rangle = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} P_{sci}(T)\,d\theta_1 d\theta_2\ ,\quad i=1,2. \qquad (17)$$

Notice that to find $\langle P_{es2}\rangle$ for GL4-256 requires $\langle P_i(T)\rangle$, i = 1 to 5. However, $\langle P_j(T)\rangle$, j = 2 to 5 are all equal since they take on the same values with the same probabilities. Thus, we need only find two averages when computing $\langle P_{es2}\rangle$.

Results and Discussion

Calculations of $\langle P_{es2}\rangle$ have been made for GL4-256 using (16) and (17), and the results are shown in Fig. 2. Similar curves for DC16-QAM can be found in [3]. For these curves, the signal-to-noise ratio is defined as

$$dBN = 10 \log\left[\frac{P}{4\sigma^2}\right] \qquad (18)$$

where P is the average power of the constellation, i.e. $P = \frac{1}{L}\sum_{I=1}^{L}\|D_I\|^2$, and the crosstalk level is given by dBI = 20 log r.

Figure 3 shows the degradation (dB) as a function of crosstalk level (dBI) for an average probability of symbol error of 10^{-6}, for both DC16-QAM and GL4-256, where the degradation is defined as the increase in dBN needed to maintain $\langle P_{es}\rangle = 10^{-6}$. As can be seen, the degradation increases with crosstalk level, making the system unusable for high crosstalk levels. By subtracting the degradation for GL4-256 from that of DC16-QAM and adding 1.24 dB, the improvement of GL4-256 over DC16-QAM can be obtained and is also shown in Fig. 3. (The 1.24 dB comes from the improvement of GL4-256 over DC16-QAM at zero-crosstalk). From Fig. 3, it is evident that for crosstalk levels below about -9.5 dB, GL4-256 performs better than or equal to DC16-QAM. However, for crosstalk levels above -9.5 dB, DC16-QAM is to be preferred. Therefore, systems which operate in mild crosstalk conditions most of the time would benefit by using GL4-256.

The probability of symbol error P_{es2} can also be obtained as a function of the crosstalk phase angles θ_1 and θ_2, by using (16). Figure 4 shows a three-dimensional plot relating those for a crosstalk level of -5 dB, and signal-to-noise ratio of 25.47 dB, which produces $\langle P_{es2}\rangle = 10^{-6}$. From Fig. 4 and the results given for DC16-QAM in [3], it is clear that P_{es} behaves similarly for both constellations. There are two lines of minimum P_{es} given by $\theta_1 = -\theta_2 + \pi$ and $\theta_1 = -\theta_2 - \pi$. There is a line of maximum P_{es} given by $\theta_1 = -\theta_2$. However, for GL4-256, P_{es} is not constant along this line, as it is for DC16-QAM, there being maximum values of P_{es} at (0,0), $(\pi/2,-\pi/2)$, $(-\pi/2,\pi/2)$, and (π,π). These coordinates represent the points at which the arguments of at least one of the complementary error functions in (16) is at a minimum value of $M_2 = (1+r^4-2r^2)/[(1+r^2+2r)2\sigma_2^2]$, where σ_2^2 is the variance given by (18) and P = 14.125. For DC16-QAM, this minimum value is $M_1 = (1+r^4-2r^2)/[(1+r^2)2\sigma_1^2]$, where σ_1^2 is the variance given by (18) and P = 20.

Consideration of M_1 and M_2 explains why $<P_{es}>$ performance for GL4-256 is better than that of DC16-QAM. For small crosstalk, $M_2 > M_1$, with the reverse being true at high crosstalk. Thus, $<P_{es2}> < <P_{es1}>$ at low crosstalk, whereas $<P_{es2}> > <P_{es1}>$ at high crosstalk, as demonstrated in Fig. 3.

For GL4-256, there are also minimum P_{es} values along the maximum P_{es} line. These occur at $(-\pi/4,\pi/4)$, $(\pi/2,-\pi/2)$, $(5\pi/4,-5\pi/4)$, and $(-5\pi/4,5\pi/4)$.

It is also of interest to compute the histogram for the same conditions used in generating Fig. 4. This is done in Fig. 5, and from this figure it can be seen that 17% of the time the instantaneous P_{es2} is higher than $<P_{es2}>$, whereas 75.7% of the time $P_{es2} \leq 0.1 <P_{es2}>$. Hence, there is a very good chance P_{es2} is better than $<P_{es2}>$ indicates, and a non-negligible time when P_{es2} is worse than $<P_{es2}>$. Since the data rate is much faster than the rate of change of the channel parameter, $<P_{es2}>$ may not be a suitable parameter to estimate the system's true performance at a given time.

Conclusion

A method has been presented to calculate the probability of symbol error for the 4D diagonalizer using arbitrary constellations. The method was applied to a 4D constellation introduced by Gersho and Lawrence [7], which was called the GL4-256 constellation in this paper. The probability of symbol error performance for GL4-256 was compared to that of DC16-QAM and it was concluded that GL4-256 performs better for low crosstalk values, i.e. below -9.5 dB, with the reverse being true for high crosstalk levels. Graphical results have been presented for the case of uniform distribution. The analytical results are general enough, however, to give more detailed information on the behavior of $<P_{es}>$ for cases when θ_1 and θ_2 are not uniformly distributed.

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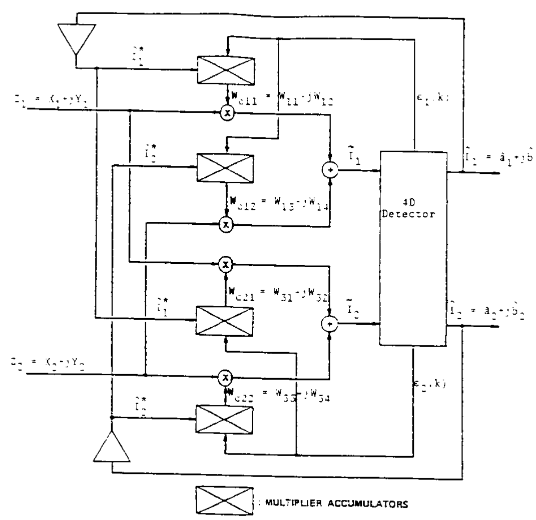
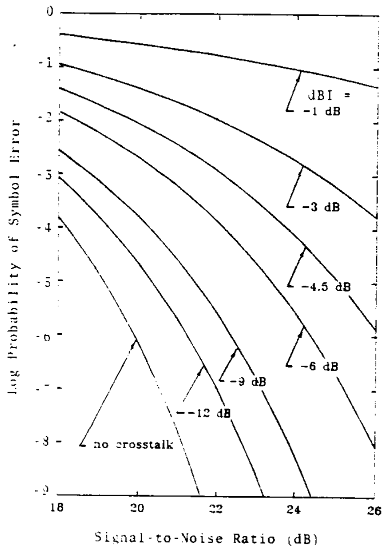


$\hat{a}_i$ is the estimate for ai , i = 1,2.

$\hat{b}_i$ is the estimate for bi , i = 1,2.

W_{ij} is the ijth entry of the W matrix.

Fig. 1. Block diagram of the adaptive 4D diagonalizer.



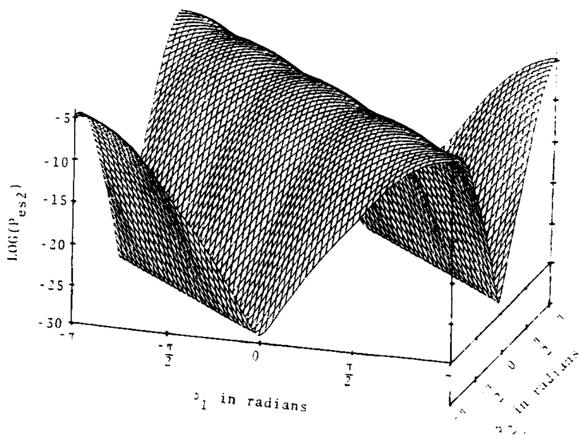
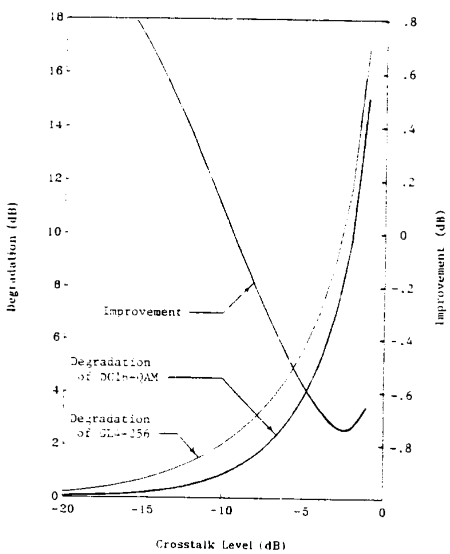


Fig. 4. Probability of symbol error P_{es2} vs. phase angles ϕ_1 and ϕ_2 for the diagonalizer with GL4-256, dBI = -5dB, and dBN = 25.47 dB. $\langle P_{es2} \rangle = 10^{-6}$.

Fig. 2. $\langle P_{es2} \rangle$ for GL4-256 vs. signal-to-noise ratio (dB).



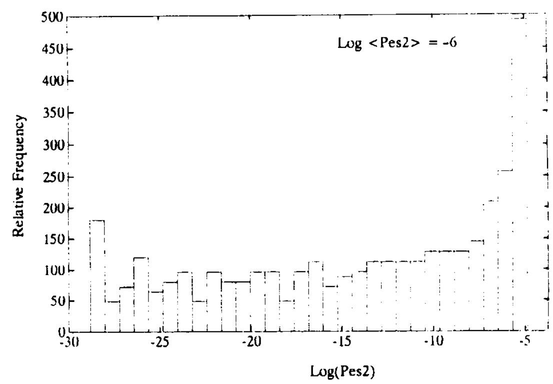


Fig. 5. Histogram showing the distribution of P_{es2} for the diagonalizer with GL4-256, for uniform distribution of ϕ_1 and ϕ_2, dBI = -5 dB, and dBN = 25.47 dB. 3,600 trials.

Fig. 3. Degradation of GL4-256 and DC16-QAM, and improvement of GL4-256 over DC16-QAM, both using the diagonalizer.

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