CONSTANT ENVELOPE EIGHT-DIMENSIONAL TRELLIS CODED MODULATION SCHEMES[1]

Edit J. Kaminsky, Kenneth V. Cartwright[2] and
Andrew B. Martinez

Tulane University
Electrical Engineering Department

ABSTRACT

This paper presents two eight-dimensional TCM schemes which use 512 points to transmit 2 b/s/Hz. The systems have constant envelope and yield asymptotic gains of 3.01 dB with only four or eight trellis states, without expanding the bandwidth over that of the equivalent uncoded system (QPSK over four consecutive time intervals). The size of the coded constellation is doubled but the MSED, the peak and average energies, and the modulation level remain identical to those of the uncoded system. The novel constellation is formed by the union of an 8D-QPSK and a rotated version of it. Our new low complexity schemes achieve small normalized redundancy, have constant envelope, and use simple and small constituent 2D constellations.

I. INTRODUCTION

Since the publication of Ungerboeck's paper [1] on Trellis Coded Modulation (TCM) much work has been done to find more and better ways to achieve improved performance by combining modulation and coding without sacrificing data rate or increasing the bandwidth of the system. In this paper we present two such schemes which use 512 points with unit energy per dimension in an eight-dimensional (8D) space.

Many applications which use nonlinear amplifiers and repeaters call for the use of constant envelope modulation schemes. Some systems, such as CPM [2], have attracted the attention of many researchers, while older systems, which include M-PSK and FSK are currently in use and have been studied extensively. The constant envelope systems we study in this paper achieve considerable gains with spectral efficiency of 2b/s/Hz if Nyquist filtering is used.

With Ungerboeck's 2D TCM schemes [1] q information bits are sent in each signaling interval using a constellation of 2^{q+1} points. Wei [3] proposed using ND constellations (N>2) with rate m/(m+1) convolutional encoders to reduce the 3 dB cost of signal expansion in Ungerboeck codes. Wei's ND scheme transmits q information bits per signaling interval using a 2L-dimensional constellation of 2^{Lq+1} points.

When ND constellations are used fewer redundant bits are added for each 2D signaling interval. The normalized redundancy of the codes presented here is $\rho=0.25$. The reduction in the size of the constituent lower dimensional constellations is also an advantage of multidimensional TCM.

The main difference between our schemes and conventional TCM systems is that the Minimum Square Euclidean Distance (MSED) between coded sequences is increased without increasing the modulation level, therefore maintaining the average energy identical to that of the uncoded base system (QPSK over four time slots).

Ungerboeck proposed in [1] the method of mapping by set partitioning to assign signal points from an expanded 2D constellation to the branches of the trellis of a convolutional encoder. Wei [3] proposed a different partitioning method for ND constellations, based on partitions of the constituent 2D constellations. We follow Wei's partitioning method to obtain the necessary subsets and then apply Ungerboeck's rules to assign them to the branches of the trellis.

Several authors [3,4,5,6] have presented 8D TCM schemes that have considerable asymptotic gains but, to the best of our knowledge, none of them achieve 3.01 dB with four or eight states, maintaining a low complexity and without increasing the modulation level. Also, most 8D schemes were used to transmit information at higher rates. An MLSE (Viterbi decoder) must be used at the receiver, but the complexity of the decoder is very reasonable.

In Section II the X8 constellation and its characteristics are presented. In Section III we follow Wei's partitioning method to obtain the necessary subsets. In Section IV we describe the rate X8-TCM systems. In section V results are presented and the systems compared to others. A summary is given in Section VI, followed in Section VII by references.

---

[1] This work was supported in part by grant No. 532148 of the Louisiana State Board of Regents.

[2] K. V. Cartwright is currently with the Technology Division, College of the Bahamas, P.O.Box N4912, Nassau, N.P., Bahamas.

CH2984-3/91/0000-0231 $1.00 © 1991 IEEE

II. THE EIGHT DIMENSIONAL CONSTELLATION X8

Figure 1 describes the system we propose to use. We form an 8D transmitted symbol by operating on eight input bits at a time. The mapper changes the input bit stream into the in-phase (I) and quadrature-phase (Q) symbol streams.

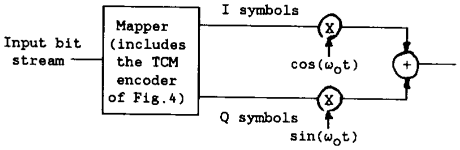

Fig.1: Block diagram of the X8 modulator.

We operate on four consecutive symbol time slots to form the transmitted symbol. If $X_0(4n+j)$ and $X_1(4n+j)$, $j=0,1,2,3$, represent the value of the I and Q data streams for the jth time slot, respectively, the i^{th} transmitted symbol can be written as $p_i = (p_{20}, p_{21}, p_{22}, p_{23})$, where p_{2j} is the 2D symbol given by $(X_0(4n+j), X_1(4n+j))$ $j=0,1,2,3$. For regular QPSK over four time slots, there are 256 unique values of p_i, since p_{2j} can be (1,1) or any sign variation of it. However, we need 512 symbols for our X8-TCM schemes, and therefore need an additional 256 symbols to allow for code redundancy. To meet the requirement of constant envelope for all time, we require that $X_0^2(4n+j) + X_1^2(4n+j) = 2$, $j=0,1,2,3$. We can fulfill both these requirements without reducing the MED of the 8D constellation or increasing the peak or average energy of the constellation. To do this we allow 8D symbols of the form $p_{ir} = (p_{20r}, p_{21r}, p_{22r}, p_{23r})$, where p_{2jr} is p_{2j} above rotated by 45 degrees. Hence, p_{2jr} can take on the values $(0,\sqrt{2})$, $(\sqrt{2},0)$, $(0,-\sqrt{2})$ and $(-\sqrt{2},0)$. Since p_{2jr} and j take on four values each, p_{ij} can be one of 4^4 unique symbols. The constellation X8 is the union of $\{p_i\}$ and $\{p_{ir}\}$.

The MED between any two symbols of the rotated 8D constellation $\{p_{ir}\}$ is two, the same as that of $\{p_i\}$. Furthermore, the MED between any symbol p_i and any symbol p_{kr} is $4+4(1-\sqrt{2})^2$. Therefore, the MED of X8 is 2, the same as that of regular QPSK over four time slots. Notice also that the peak and average energy of X8 is 8. Hence, we can transmit 512 symbols with the same energy that regular 8D-QPSK needs for only 256 symbols. We see that the additional points in the constellation we use are at the same level as the points used in the uncoded system; this is what makes our system more attractive than others.

III. PARTITION OF THE X8 CONSTELLATION

Several partitioning methods are available to us. Since X8 is constituted by two 2D QPSK constellations, we chose to follow Wei's method [3]. We start by partitioning each constituent QPSK constellation (2D lattice[3]) into four 2D sublattices (A, B, C and D) of one point each, as shown in Fig. 2:

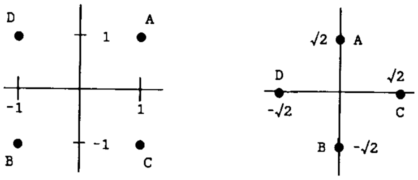

Fig.2: Set partitioning of the constituent QPSK constellations.

If the QPSK constellations are thought of as being part of the lattice Z^2 [5] the MSED between the sublattices is easily shown to be 16. Now we form sixteen 4D types for each of the two constituent 2D sublattices by concatenating a pair of 2D points:

(A,A), (A,B), (A,C), (A,D)
(B,A), (B,B), (B,C), (B,D)
(C,A), (C,B), (C,C), (C,D)
(D,A), (D,B), (D,C), (D,D)

From these sixteen types we construct eight 4D sublattices (for each of the two groups) by grouping together two types such that the MSED of the 4D sublattices is still 16:

0: (A,A), (B,B)
1: (C,C), (D,D)
2: (A,B), (B,A)
3: (C,D), (D,C)
4: (A,C), (B,D)
5: (C,B), (D,A)
6: (A,D), (B,C)
7: (C,A), (D,B)

We proceed to form two groups of sixty-four 8D types each, by concatenating elements from the above 4D types:

(0,0), (0,1), ..., (0,7)
(1,0), (1,1), ..., (1,7)
...
(7,0), (7,1), ..., (7,7)

We further group the 64 8D types from each group into 16 8D sublattices with MSED 16. We denote these by S_0 through S_{15} for the group with coordinates ±1, and S_{16} through S_{31} for the group with coordinates $(0,\pm\sqrt{2})$ or $(\pm\sqrt{2},0)$:

S_0 or S_{16}: (0,0), (1,1), (2,2), (3,3)
S_1 or S_{17}: (0,1), (1,0), (2,3), (3,2)
S_2 or S_{18}: (0,2), (1,3), (2,0), (3,1)
S_3 or S_{19}: (0,3), (1,2), (2,1), (3,0)
S_4 or S_{20}: (4,4), (5,5), (6,6), (7,7)
S_5 or S_{21}: (4,5), (5,4), (6,7), (7,6)
S_6 or S_{22}: (4,6), (5,7), (6,4), (7,5)
S_7 or S_{23}: (4,7), (5,6), (6,5), (7,4)
S_8 or S_{24}: (0,4), (1,5), (2,6), (3,7)
S_9 or S_{25}: (0,5), (1,4), (2,7), (3,6)
S_{10} or S_{26}: (0,6), (1,7), (2,4), (3,5)
S_{11} or S_{27}: (0,7), (1,6), (2,5), (3,4)
S_{12} or S_{28}: (4,0), (5,1), (6,2), (7,3)
S_{13} or S_{29}: (4,1), (5,0), (6,3), (7,2)

---

[3] The constellation formed by $(\pm\sqrt{2},0)$ and $(0,\pm\sqrt{2})$ is not strictly a lattice, but we will refer to it as one to maintain Wei's notation.

S_{14} or S_{30}: (4,2), (5,3), (6,0), (7,1)
S_{15} or S_{31}: (4,3), (5,2), (6,1), (7,0)

Finally, we must form the subsets to be assigned to the trellis. Since we will use convolutional encoders of rates 1/2 and 2/3, we need 4 and eight subsets, respectively. The subsets are formed in such a way that the intrasubset MSED is reduced to eight. We call the subsets W_1, X_1, Y_1 and Z_1 for the first case, and R_2, T_2, U_2, V_2, W_2, X_2, Y_2 and Z_2 for the second. The eight subsets which will be used for the rate 2/3 TCM scheme are

$R_2 = \{S_0,S_1,S_2,S_3\}$ $W_2 = \{S_{16},S_{17},S_{18},S_{19}\}$
$T_2 = \{S_4,S_5,S_6,S_7\}$ $X_2 = \{S_{20},S_{21},S_{22},S_{23}\}$
$U_2 = \{S_8,S_9,S_{10},S_{11}\}$ $Y_2 = \{S_{24},S_{25},S_{26},S_{27}\}$
$V_2 = \{S_{12},S_{13},S_{14},S_{15}\}$ $Z_2 = \{S_{28},S_{29},S_{30},S_{31}\}$

and the four for the rate 1/2 scheme are

$W_1 = \{R_2,T_2\}$ $X_1 = \{U_2,V_2\}$
$Y_1 = \{W_2,X_2\}$ $Z_1 = \{Y_2,Z_2\}$.

IV. THE TCM SCHEMES

1. THE TCM SCHEME OF RATE 1/2

This TCM system consists of a convolutional encoder of rate $R_c=1/2$ and a QPSK modulator over four consecutive time intervals. One of the eight information bits arriving every four signaling intervals (I_7) enters the convolutional encoder and two coded bits are produced at its output (O_7 and O_8); these two coded bits select one of the four subsets W_1, ..., Z_1. Three uncoded bits (I_6, I_5 and I_4) select the sublattice from within the subset, and the remaining uncoded bits (I_3 through I_0) select, from within the selected sublattice, the point that will be transmitted. The system is shown in Fig. 3.

The convolutional encoder chosen, shown in Fig. 4, is of the feedback type, has rate $R_c=1/2$ and constraint length $\nu=2$. The trellis diagram is shown in Fig. 5, where the states are named according to the contents of the memory cells and the subsets associated with the transitions diverging from each state are shown beside the respective states in a top down fashion.

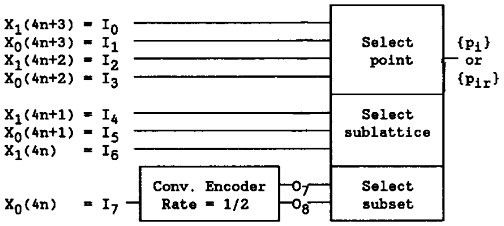

Fig.3: Block diagram of the rate 1/2 TCM scheme.

The idea behind the assignment of points to branches of the trellis is to maximize the free distance of the code. In our case this distance is 8, the MSED between points in any of the 8D subsets. Minimization of the error coefficient was also considered. The assignment satisfies the following rules of mapping by set partitioning [1]: 1) Points with maximum MSED are assigned to parallel transitions, and 2) The 8D sublattices associated with transitions diverging from, or merging into a trellis state, are different from each other and have the following maximum ED.

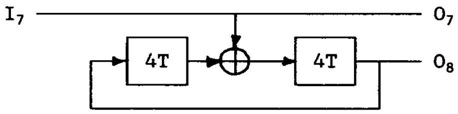

Fig. 4: Feedback convolutional encoder of $R_c=1/2$ and $\nu=2$.

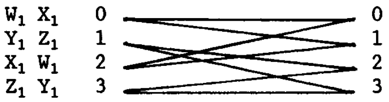

Fig. 5: Trellis diagram of the rate 1/2 TCM scheme. Each line represents 128 parallel transitions.

2. THE TCM SCHEME OF RATE 2/3

This TCM system consists of a convolutional encoder of rate $R_c=2/3$ and, again, a QPSK modulator over four consecutive time intervals. Two bits (I_7 and I_6) enter, and three bits (O_6, O_7 and O_8) leave, the convolutional encoder. The three coded bits select one of the eight subsets R_2, T_2, U_2 ..., Z_2. Two uncoded bits (I_5 and I_4) select the sublattice from within the subset, and the remaining uncoded bits (I_3 through I_0) select, from within the selected sublattice, one of the sixteen points that will be transmitted. The overall system is shown in Fig. 6, the rate 2/3 feedback convolutional encoder in Fig. 7, and the trellis diagram in Fig. 8.

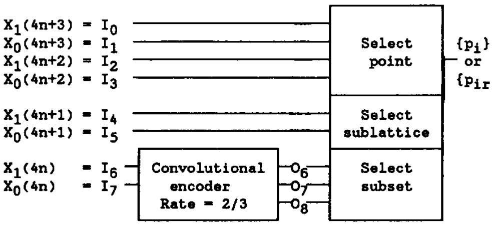

Fig.6: Block diagram of the rate 2/3 TCM scheme.

V. RESULTS

The MSED between any pair of paths for the systems presented in the last section is 8 due to the parallel transitions; This is the free

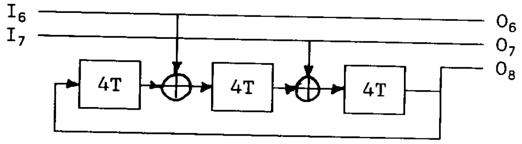

Fig. 7: Convolutional encoder of rate $R_c=2/3$.

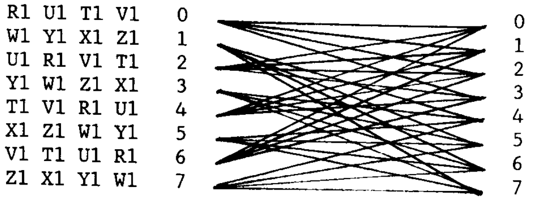

Fig. 8: Trellis diagram of the rate 2/3 TCM scheme. Each line represents 64 parallel branches.

squared distance of the code. The MSED of the uncoded 8D QPSK system needed to transmit 2 b/s/Hz is $d_u^2 = 4$. The average (and peak) energy of both coded and uncoded systems is E = 8. The asymptotic gain of the X8 system is therefore

$$G_a = 10\log_{10}\left(\frac{d_{free}^2}{d_u^2}\right) = 10\log_{10}\left(\frac{8}{4}\right) = 3.01\ dB \tag{1}$$

To make fair comparisons between different trellis codes, one needs to take into account the error coefficient, the complexity of the code and the modulation level. Originally, the number of states (2^ν) was used to determine the realization expense of the Viterbi Decoder. Later, Wei proposed in [3] the ratio of the total number of allowed state transitions of a code to the number of signaling intervals, N_T, as a measure of code complexity, β. Our rate 1/2 code has an extremely low complexity of $\beta = 8/4 = 2$, while the rate 2/3 code's complexity is $\beta = 32/4 = 8$. Both schemes have a modulation level of 2.

The error coefficient normalized per two dimensions, N_o, is the average number of sequence paths at MSED (N_{free}), divided by the number of 2D signaling intervals (N_T). For the rate 1/2 code presented in this paper we have 28 paths of length one (parallel transitions) at d_{free}; the coded normalized error coefficient is therefore, $N_{o,c} = (28)/4 = 7$, while the normalized error coefficient of the base uncoded constellation is $N_{o,u} = 8/4 = 2$. The rate 2/3 code has 12 parallel paths at d_{free}, yielding $N_{o,c} = 12/4 = 3$.

We use Forney's rule which states that every doubling of the error coefficient with respect to that of the uncoded system causes a loss of 0.2 dB [4]. The loss, λ, of our rate 1/2 X8-TCM system is therefore

$$\lambda = \frac{\log\left(\frac{N_{o,c}}{N_{o,u}}\right)}{\log(32)} = \frac{\log\left(\frac{7}{2}\right)}{\log(32)} = 0.36\ dB \tag{2}$$

while for the rate 2/3 code it is

$$\lambda = \frac{\log\left(\frac{3}{2}\right)}{\log(32)} = 0.11\ dB \tag{3}$$

The effective gains of the X8-TCM schemes are thus $\gamma_{eff} = G_a - \lambda = 3.01 - 0.36 = 2.65$ dB, and $\gamma_{eff} = 3.01 - 0.11 = 2.9$ dB, for the codes of rate 1/2 and 2/3, respectively. The error coefficient can be reduced by increasing the number of trellis states, thus trading complexity for a higher effective gain. The redundancy of X8-TCM is 1 bit per 8D, which yields a normalized redundancy per 2D, ρ, of 1/4.

The phase transparency of the code becomes important when the system is susceptible to phase rotations. Each sublattice S_0-S_{31} used in the X8-TCM scheme is invariant under 90° rotations, i.e. if each 2D constituent point in lattice S_i is rotated by 90°, another 8D point in S_i is obtained, $i=0,1,\ldots,31$. This means that X8-TCM can easily be made invariant under rotations of integer multiples of 90°, by using differential encoding/decoding [3].

Table I summarizes the characteristics of the TCM schemes presented in this paper and other TCM systems of rate 2 bit/T with the same number of trellis states. The reference uncoded system is QPSK over N_T signaling intervals, except where indicated. C_c denotes the constellation or modulation used for the coded system, and ML its modulation level. It is clear from table I that although our systems do not achieve outstanding gains, they yield quite low complexities β (the smallest of all those codes considered here), low redundancy ρ, and the lowest modulation level, and are therefore good alternatives to conventional TCM systems.

In general, those codes with asymptotic and effective gains larger to that of X8 usually suffer from one or more of the following: higher complexity, large constituent 2D constellations, high redundancy, constellation asymmetry and increased modulation level. Our X8-TCM systems avoid all these disadvantages while yielding conservative gains and maintaining a constant envelope, all with only four or eight trellis states.

VI. SUMMARY AND SUGGESTIONS

New 8D TCM schemes, called X8-TCM, which use a constellation of 512 points with unit energy per dimension in combination with simple convolutional encoders and a QPSK modulator over four signaling intervals were presented in this paper. It has been shown that the increase in MSED between coded sequences using these schemes yield asymptotic gains of 3.01 dB without sacrificing data rate or increasing bandwidth. With the new schemes there is no constellation expansion penalty paid due to the efficient use of the 8D space. Also, the normalized redundancy is only 1/4 and the complexity is only 8 or less. The X8-TCM system also has constant envelope and can be made invariant under rotations of 90°.

The gain of X8-TCM could be increased to 6.02 dB by increasing the number of trellis states and, thus, the systems' complexity. Simulations could be performed to more accurately evaluate the actual performance of the system presented, in terms of probability of error vs. SNR. Constellations similar to the one used here can be formed in other dimensions and used to design new TCM schemes with nice characteristics.

VII. REFERENCES

[1] Ungerboeck, G., "Channel coding with multilevel/ phase signals," *IEEE Trans. Info. Th.*, vol. IT-28, pp.55-67, Jan. 1982.

[2] Aulin, T. and C. Sundberg, "Continuous phase modulation -Parts I and II," *IEEE Trans. Commun.*, vol. COM-29, pp. 196-225, 1981.

[3] Wei, L., "Trellis-coded modulation with multidimensional constellations," *IEEE Trans. Info. Th.*, vol.IT-33, pp. 483-501, Jul. 1987.

[4] Forney, G., "Coset codes. Part I: Introduction and geometrical classification. *IEEE Trans. Info. Th.*, vol.43, pp. 1123-1151, Sep. 1988.

[5] Pietrobon, S. et al., "Trellis-coded multidimensional phase modulation," *IEEE Trans. Info. Th.*, vol. 36, pp. 63-89, Jan. 1990.

[6] Calderbank, A. and N. Sloane, "An eight-dimensional trellis code," *Proc. of the IEEE*, vol. 74, pp.757-759, May 1986.

[7] Padovani, R. and J. Wolf, "Coded phase/-frequency modulation," *IEEE Trans. commun.*, vol. COM-34, pp. 446-453, May 1986.

[8] Divsalar, D., M. Simon and J. Yuen, "Trellis coding with asymmetric modulations," *IEEE Trans. Commun.*, vol. COM-35, pp. 130-141, Feb. 1987.

[9] Ungerboeck, G., "Trellis-coded modulation with redundant signal sets, Parts I and II," *IEEE Commun. Mag.*, vol. 25, pp. 5-21, Feb. 1987.

TABLE I
Comparison of TCM schemes of rate 2bit/T.

N	N_T	C_c	ML	ν	R_c	ρ	G_a(dB)	N_o	β	$\gamma_{eff}^{(1)}$	Ref.
8	4	X8	4	2	1/2	1/4	3.01	7	2	2.65	Here
8	4	X8	4	3	2/3	1/4	3.01	3	8	2.89	Here
2	1	8-PSK	8	2	1/2	1	3.01	1	8	3.01	[1]
2	1	8-PSK	8	3	2/3	1	3.60	2	32	3.60	[1]
2	1	8AM-PM	8	2	1/2	1	2.00	-	8	-	[1]
2	1	8AM-PM	8	3	2/3	1	3.01	-	32	-	[1]
4	1	2FSK/QPSK	4	2	1/2	1/2	3.01	-	8	-	[7]
4	1	2FSK/QPSK	4	3	2/3	1/2	3.7-4.8$^{(2)}$	-	32	-	[7]
1	1	8-PAM$^{(3)}$	8	2	1/2	2	3.31	8	8	3.11	[9]
1	1	8-PAM$^{(3)}$	8	3	1/2	2	3.77	8	16	3.57	[9]
1	1	Asy8-PAM$^{(3)}$	8	2	2/3	2	3.55	-	16	-	[8]
1	1	Asy8-PAM$^{(3)}$	8	3	2/3	2	4.15	-	32	-	[8]
4	2	8-PSK	8	2	1/2	1/2	3.01	3	4	2.89	[5]
4	2	8-PSK	8	3	2/3	1/2	3.01	1	16	3.01	[5]
6	3	8-PSK	8	2	1/2	1/3	3.01	5	3	2.75	[5]
6	3	8-PSK	8	3	2/3	1/3	3.01	2.3	11	2.97	[5]
8	4	8-PSK	8	2	2/3	1/4	3.01	3	4	2.89	[5]
8	4	8-PSK	8	3	3/4	1/4	3.01	1	16	3.01	[5]

(1) This quantity is obtained either by using the rule of 0.2dB loss every doubling of N_o or by curves given in reference for SNR at pe of 10^{-6}.

(2) Gains are dependent on the frequency modulation index h. Bandwidth is slightly changed.

(3) The reference system is uncoded 4-PAM.