Blind Phase Recovery in General QAM Communication Systems Using Alternative Higher Order Statistics

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***Abstract*— Blind phase recovery for square QAM communication systems using higher order statistics is well established. It is demonstrated that this method can be adapted for cross QAM constellations, if alternative fourth-order statistics are used. However, a four quadrant inverse tangent function must now be used. Monte Carlo simulation provides evidence of the usefulness of the approach.**

***Index Terms*—Carrier phase estimation, higher order statistics, quadrature amplitude modulation, synchronization.**

I. Introduction

CHEN *et al.* [1] have introduced a method of blind phase recovery for square QAM communication systems using higher order statistics. Their method assumed independence of the in-phase and quadrature-phase components. Therefore, their method cannot be expected to work for 32-QAM, or any nonsquare QAM system. The purpose of this letter is to point out that their method can be modified to work for nonsquare (i.e., cross) QAM, by relaxing the requirement that the in-phase and quadrature-phase components be independent, and by using an alternative fourth order statistic, and a four quadrant inverse tangent function. One advantage of this modification is that it is not necessary to know the order of the transmitted QAM system, unlike the method of Chen *et al.*

II. Statement of the Problem

The received signal is given by

$$Y = e^{j\theta}X + N \tag{1}$$

where X is the complex transmitted symbol, $N = N_r + jN_i$ is complex Gaussian noise, and θ is the phase angle to be determined by observing the received signal Y. It is further assumed that X_r and X_i in $X = X_r + jX_i$ are uncorrelated non-Gaussian random variables with identical and symmetrical distributions, and that they satisfy (6). Note that Chen *et al.* [1] assumed X_r and X_i were independent.

The received signal can be rewritten in terms of the received in-phase Y_r and quadrature-phase Y_i components as follows:

$$Y_r = X_r\cos\theta - X_i\sin\theta + N_r$$

$$Y_i = X_r\sin\theta + X_i\cos\theta + N_i. \tag{2}$$

The blind detection problem is to find an estimate for θ, without actually detecting the data X. Note that because X has quadrant symmetry, it is only possible to recover θ within 90°.

III. Modification of the Method of Chen *et al.*

A. Review of the Method of Chen et al.

Chen *et al.* provide the following method based upon higher order statistics.

Step 1: Define the fourth-order cumulants across Y_r and Y_i as

$$\begin{aligned}\gamma &= E[Y_r^2Y_i^2] - E[Y_r^2]E[Y_i^2] - 2\{E[Y_rY_i]\}^2,\\ \gamma_a &= E[Y_r^3Y_i] - 3E[Y_r^2]E[Y_rY_i],\\ \gamma_b &= E[Y_rY_i^3] - 3E[Y_i^2]E[Y_rY_i].\end{aligned} \tag{3}$$

Under the assumption of independence, Chen *et al.* state that

$$\begin{aligned}\gamma &= 0.25\gamma_x\sin^2 2\theta,\\ \gamma_a - \gamma_b &= 0.5\gamma_x\sin 2\theta\cos 2\theta,\\ &= 0.25\gamma_x\sin 4\theta, \quad \text{where}\\ \gamma_x &= E[|X|^4] - 2\{E[|X|^2]\}^2.\end{aligned} \tag{4}$$

Step 2: Solve for θ using the following:

$$\begin{aligned}\cot 2\theta &= \frac{\gamma_a - \gamma_b}{2\gamma} \quad \text{if } \left|\frac{\gamma}{\gamma_x}\right| \geq 0.125\\ \tan 2\theta &= \frac{2(\gamma_a - \gamma_b)}{\gamma_x - 4\gamma} \quad \text{if } \left|\frac{\gamma}{\gamma_x}\right| < 0.125.\end{aligned} \tag{5}$$

B. Modification of the Method

To modify the method, independence of the in-phase and quadrature-phase components is no longer assumed. Instead, the following assumptions about X are made:

$$\begin{aligned}E[X_rX_i] &= 0\\ E[X_r] &= E[X_i] = 0\\ E[X_r^2] &= E[X_i^2]\\ E[X_r^3X_i] &= E[X_i^3X_r].\end{aligned} \tag{6}$$

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TABLE I
ESTIMATES OF θ IN LOWER SIGNAL-TO-NOISE

True θ	QPSK N=64 SNR$_b$=6 dB		16-QAM N=200 SNR$_b$=10 dB		32-QAM N=4000 SNR$_b$=13 dB		64-QAM N=512 SNR$_b$=15 dB		128-QAM N=4000 SNR$_b$=18 dB	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
θ =0°	-0.098	2.282	0.002	1.487	0.040	1.684	-0.014	1.054	-0.004	1.617
θ =15°	15.061	2.277	15.016	1.446	15.016	1.719	14.963	1.026	14.902	1.731
θ =30°	29.877	2.336	30.097	1.431	29.943	1.738	29.913	1.075	30.039	1.866
θ =45°	45.023	2.276	44.968	1.430	44.964	1.714	45.125	1.154	44.995	1.828
θ =60°	-29.971	2.212	-29.953	1.387	-30.004	1.602	-29.950	1.060	-30.005	1.800
θ =75°	-14.945	2.357	-15.036	1.400	-15.052	1.666	-15.036	1.135	-14.863	1.790
θ =90°	0.043	2.293	-0.047	1.390	0.003	1.702	0.041	1.083	-0.046	1.799

TABLE II
ESTIMATES OF θ IN HIGHER SIGNAL-TO-NOISE

True θ	QPSK N=64 SNR$_b$=10 dB		16-QAM N=200 SNR$_b$=15 dB		32-QAM N=4000 SNR$_b$=17 dB		64-QAM N=512 SNR$_b$=20 dB		128-QAM N=4000 SNR$_b$=22 dB	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
θ =0°	0.052	1.324	-0.045	1.207	0.036	1.641	-0.026	1.066	0.000	1.612
θ =15°	15.082	1.245	15.009	1.198	15.014	1.667	15.042	1.067	14.905	1.726
θ =30°	29.893	1.261	30.028	1.185	29.940	1.703	30.025	0.986	30.037	1.864
θ =45°	45.004	1.301	44.886	1.125	44.965	1.667	44.982	1.047	44.993	1.815
θ =60°	-30.023	1.179	-29.951	1.219	-30.014	1.555	-29.939	1.103	-30.000	1.791
θ =75°	-15.047	1.238	-14.973	1.223	-15.056	1.627	-15.078	1.077	-14.861	1.779
θ =90°	-0.015	1.242	-0.020	1.159	-0.011	1.641	-0.015	1.071	-0.042	1.792

In addition, the following fourth-order statistic is used instead of γ in (3):

$$\gamma = E[Y_r^4] + E[Y_i^4] - 6E[Y_r^2 Y_i^2]. \quad (7)$$

With the assumptions of (6), it is straightforward, but tedious to show that (7) becomes

$$\gamma = \gamma_x \cos 4\theta, \quad \text{where}$$
$$\gamma_x = E[X_r^4] + E[X_i^4] - 6E[X_r^2 X_i^2]. \quad (8)$$

Furthermore, $\gamma_a - \gamma_b$ is still given by (3) and (4), provided γ_x of (4) is replaced by γ_x of (8). [Note that γ_x of (8) is equal to γ_x of (4) if the components of X are independent].

It should now be clear that an estimate of θ can be found from

$$\tan 4\theta = 4\left(\frac{\gamma_a - \gamma_b}{\gamma}\right). \quad (9)$$

It is fortunate that (9) does not produce a biased estimate when noise is present. This fact is easy to verify, but tedious.

As it is desired to know θ within $\pm 45°$, the four quadrant inverse tangent has to be used to calculate θ from (9). Note that it is not necessary to know γ_x to determine θ, and therefore, it is not necessary to know what constellation is being transmitted. This is an advantage for multiconstellation systems. However, γ_x is negative for the M-QAM systems considered in this letter, thereby making the right had side of to be

$$\frac{-\sin 4\theta}{-\cos 4\theta}.$$

Hence

$$\theta = \frac{1}{4}\tan^{-1}\left(\frac{-4(\gamma_a - \gamma_b)}{-\gamma}\right).$$

Indeed, this is the equation that was used in the simulations of Tables I and II and of the following section (with the parameters replaced by their estimates, of course).

Furthermore, it is not difficult to demonstrate that M-PSK and nonsquare M-QAM constellations satisfy the assumptions made in (6). Unfortunately, however, this modification will not work for M-PSK, as the author has found numerically that $\gamma_x = 0$, and hence (9) would be indeterminate. Therefore, the method of Chen *et al.* is extended to include cross QAM constellations.

IV. MONTE CARLO SIMULATION

To demonstrate that the proposed extension does indeed work, computer simulation results will now be presented. As in [1], each simulation result is obtained from 500 Monte Carlo runs with different data length N. Table I shows the estimate of θ for low bit signal-to-noise (SNR_b) ratio, where the probability of symbol error is in the range of $P_M = 10^{-2} - 10^{-3}$. Table II, on the other hand, gives similar results for higher SNR_b when $P_M < 10^{-5}$.

Comparing Tables I and II of this report to those in [1], it can be seen that this modification works as well or better than the method in [1] for square QAM. Notice that the standard deviation appears to be fairly constant with respect to the true value of θ. This is unlike that of [1], where the standard deviation appears to depend on θ. For example, the standard deviation is consistently higher for $\theta = 15^\circ$ or $\theta = 75^\circ$, for all constellations and SNR_b.

For cross constellations, Tables I and II clearly indicate larger values of N are needed. This is also the case for cross constellations when the fourth power phase estimator is used [2].

V. Conclusion

In this letter, a method to blindly estimate phase for general M-QAM systems using higher order statistics has been presented. The method does not require knowledge of the order of the system. Monte Carlo simulations have provided the experimental verification of this approach.

References

[1] L. Chen, H. Kusaka, and M. Kominami, "Blind phase recovery in QAM communication systems using higher order statistics," *IEEE Signal Processing Lett.*, vol. 3, pp. 147–149, May 1996.

[2] C. N. Georghiades, "Blind carrier phase acquisition for QAM constellations," *IEEE Trans. Commun.*, vol. 45, pp. 1477–1486, Nov. 1997.