A Comparison of Three Diagonalizers, Adaptive Crosstalk Cancellers, in Dual-Polarized M-QAM Systems

S. T. Hsieh, K. V. Cartwright, P. F. Duvoisin, and E. P. Williamson

Abstract—**Three different adaptive diagonalizers are compared on a probability of symbol error performance basis for dual-channel M-QAM systems. One diagonalizer ($D3$) greatly outperforms the other two, and any comparison of the performance of the diagonalizers to that of the minimum mean square error (MMSE) canceller should be based on $D3$. Receiver structures are also presented.**

I. Introduction

The diagonalizer and the minimum mean square error (MMSE) canceller are two adaptive receiver structures that reduce the problem of depolarization crosstalk in dually polarized channels, and these have been studied by many authors. In [1], Kavehrad compared their probability of symbol error performance and concluded that the MMSE receiver was substantially better than the diagonalizer. However, the diagonalizer studied in [1], hereafter called the $D1$ diagonalizer, is not the only diagonalizer found in the literature. In fact, we have identified two other diagonalizers. One of these, hereafter called the $D2$ diagonalizer, was proposed by Kavehrad *et al.* [2] as a normalized version of the $D1$ diagonalizer. The other was analyzed by Duvoisin *et al.* [3], and will be called the $D3$ diagonalizer.

The purpose of this correspondence is to compare these three diagonalizers on a probability of symbol error performance basis when dual-channel M-QAM is transmitted. It will be shown that $D3$ performs best, followed by $D2$ and, as expected, the $D1$ diagonalizer performs worst of all. Receiver structures for all three diagonalizers will be presented.

II. Dual-Channel M-QAM System

The dual-channel M-QAM model is well documented in [1]–[4], and we will review it only very briefly here.

A transmitted vector (symbol) can be written as $D = [(a_1 + jb_1), (a_2 + jb_2)]^T$, with superscript T indicating the transpose of the vector, a_1, b_1, a_2, and b_2 are data sequences which take on values with equal probability from the set $\{\pm 1, \pm 3, \cdots, \pm (L-1)\}$ with $L = M^{1/2}$. The a_i sequence modulates the in-phase carrier of channel i, whereas b_i modulates the quadrature phase of channel i where $i = 1, 2$.

The channel is assumed to be slowly varying and nondispersive. Thus, the model is applicable to the satellite path which takes on new significance for the dual-channel M-QAM case, since it has been recently demonstrated by Feher [5] that M-QAM systems are possible over the satellite path, at least for $M = 16$.

According to Amitay [6], the model is also applicable to terrestrial radio systems in the 4 GHz band for fades which do not exceed 30 dB. In addition, the channel corrupts the transmission by introducing a fraction of one information stream into the other. Also, zero-mean AWGN is present with variance σ^2 in each component of both channels. Thus, the received symbol can be written as

$$Z = TD + N \tag{1}$$

where Z is the received symbol, i.e., $Z = [(x_1 + jy_1), (x_2 + jy_2)]^T$, with x_i as the received sequence in the in-phase demodulator of channel i, and y_i is the received sequence in the quadrature demodulator of that same channel, $i = 1, 2$.

N is the noise vector $[(N_{1R} + jN_{1I}), (N_{2R} + jN_{2I})]^T$. T is the channel matrix used in [1] and defined as

$$T = \begin{bmatrix} 1 & re^{j\phi_1} \\ re^{j\phi_2} & 1 \end{bmatrix}. \tag{2}$$

III. Probability of Symbol Error Performance for the Diagonal Receiver

The conventional QAM detector operates on the received symbol Z to make four individual one-dimensional decisions to arrive at an estimate for D. The diagonalizer, however, first multiplies Z by another matrix W_J ($J = 1, 2, 3$) to give the diagonalized symbol

$$Z_{DJ} = W_J Z = W_J TD + W_J N. \tag{3}$$

The symbol Z_{DJ} is then operated on by the conventional QAM detector. The diagonalizers differ only in the choice of the weight matrix with the Di diagonalizer using the W_i matrix, $i = 1, 2, 3$ where $W_1 = (1 - r^2 e^{j\phi})T^{-1}$, $W_2 = W_1/(1 - r^2 \cos\phi)$, $W_3 = T^{-1}$, and $\phi = \phi_1 + \phi_2$. Implementations for each of these are given in Section V.

It is straightforward to show that the probability that $\hat{a}_1$, the estimate for a_1, is in error, for the Di diagonalizer, is given by [2], [3]

$$P_i(e \mid \phi) = \frac{L-1}{L} E\left[\operatorname{erfc}\left(\frac{1 - \Delta_i}{\sqrt{2}\,\sigma k_i}\right)\right], \quad i = 1, 2, 3 \tag{4}$$

where $\Delta_1 = -a_1 r^2 \cos\phi + b_1 r^2 \sin\phi$, $\Delta_2 = b_1 r^2 (\sin\phi)/(1 - r^2\cos\phi)$, $\Delta_3 = 0$, $k_1^2 = (1 + r^2)$, $k_2^2 = (1 + r^2)/(1 - r^2 \cos\phi)^2$, $k_3^2 = (1 + r^2)/(1 + r^4 - 2r^2\cos\phi)$, and the expectation is with respect to the data sequences a_1 and b_1. The probability that $\hat{a}_1$, b_1, $\hat{a}_2$, or b_2 is in error, that is, a symbol is in error, is approximately given by

$$P_{es}(\phi) \simeq 4P_i(e \mid \phi), \quad i = 1, 2, 3 \tag{5}$$

The Δ_1 term greatly reduces the performance of the $D1$ diagonalizer, since it causes the argument of the complementary error function to be reduced. This effect will be more of a problem as the level M of the dual-channel M-QAM system is

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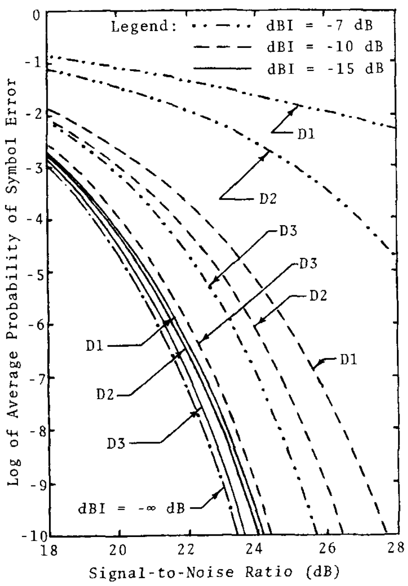


Fig. 1. $\langle P_{es} \rangle$ for dual-channel 16-QAM.

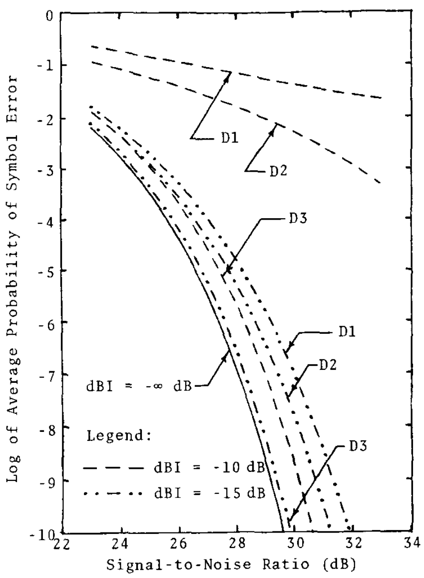


Fig. 2. $\langle P_{es} \rangle$ for dual-channel 64-QAM.

increased. The $D2$ diagonalizer reduces this problem somewhat by removing the a_1 dependence by normalizing the a_1 decision variable of $D1$ by dividing by $(1 - r^2 \cos \phi)$ [2]. On the other hand, $D3$ totally eliminates the Δ term, which means its performance does not depend upon the level M of the system, other than through the $(L - 1)/L$ factor, and (7).

If it is assumed, as is usually done, that ϕ_1 and ϕ_2 are uniformly distributed from $-\pi$ to π, one may also define the average probability of symbol error $\langle P_{es} \rangle$ as

$$\langle P_{es} \rangle = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} P_{es}(\phi)\, d\phi_1\, d\phi_2. \qquad (6)$$

IV. Results and Discussion

Calculations of $\langle P_{es} \rangle$ have been made using (4), (5), and (6) for dual-channel 16-QAM shown in Fig. 1, and for dual-channel 64-QAM shown in Fig. 2. The performance of $D2$ in Fig. 1 agrees with Fig. 9 of [2] if allowance is made for the difference in the definition of symbol error, i.e., [2] uses (4) as the probability of symbol error, whereas we use (5). For these graphs, the signal-to-noise (dBN) ratio is defined as

$$\text{dBN} = 10 \log \left(\frac{L^2 - 1}{3\sigma^2} \right) \qquad (7)$$

where $L = 4$ for dual-channel 16-QAM, and $L = 8$ for dual-channel 64-QAM. Also, the crosstalk level is given by dBI $= 20 \log r$.

From Figs. 1 and 2, it is clear that $D3$ vastly outperforms $D2$ which outperforms $D1$. The performance differences become more important as the crosstalk level increases. To further illustrate this, a graph of degradation versus crosstalk level for a $\langle P_{es} \rangle$ of 10^{-6} is shown in Fig. 3. By degradation, we mean the amount the signal-to-noise has to increase in order to maintain the average probability of symbol error at a level of 10^{-6}. From Fig. 3, it is seen that the degradation does not depend upon the level M of dual-channel M-QAM being transmitted for the $D3$ diagonalizer. This is not the case for $D1$ and $D2$ where the performance decreases considerably in going from dual-channel 16-QAM to dual-channel 64-QAM. (It can be shown that this behavior is directly due to the Δ_i term in (4)). Thus, we conclude that $D3$ is the preferred diagonalizer, and any comparison between the MMSE canceller and the diagonalizer should be based on the $D3$ diagonalizer's performance. This was done by Duvoisin *et al.* [4] for dual-channel 16-QAM where it was concluded that from a practical point of view the differences are trivial for the signal-to-noise ratios that are normally of interest. This can be demonstrated analytically by realizing that the weight matrix for the MMSE is given by

$$W_{\text{MMSE}} = T^H \left(\frac{\sigma^2}{P} I + TT^H \right)^{-1} \qquad (8)$$

where $P = E(a_1^2) = E(b_1^2) = E(a_2^2) = E(b_2^2)$, and superscript H indicates the conjugate transpose of the vector. The diagonal elements of TT^H are all equal to $1 + r^2$. Hence, if $\sigma^2/P \ll 1 + r^2$, then $W_{\text{MMSE}} \rightarrow T^{-1} = W_3$. Clearly, then for large signal-to-noise ratios, the $D3$ diagonalizer performance is approximately equal to that of the MMSE canceller. This is the same conclusion reached by Nichols *et al.* in [7] for the dual-channel QPSK case.

V. Receiver Structures

We have shown above that $D3$ is the preferred diagonalizer and the natural question which arises is, "How is this to be implemented?" Ironically, the answer is supplied by Kavehrad [1]. The receiver structure given by him in [1], and shown here in Fig. 4, switch in position A, actually implements $D3$ and not $D1$ which is the diagonalizer he analyzed in that paper. In fact, $D1$ is implemented by requiring W_{11} and W_{22} to be unity, as in switch position B, in Fig. 4. In this case, W_{21} converges to $-T_{21}$, and W_{12} converges to $-T_{12}$. Thus, the channel parameters are readily available from the $D1$ and $D2$ diagonalizers. The analysis done in [3] supports these claims. Finally, Kavehrad *et al.* [2] suggested the normalization of $D1$ to form $D2$ by

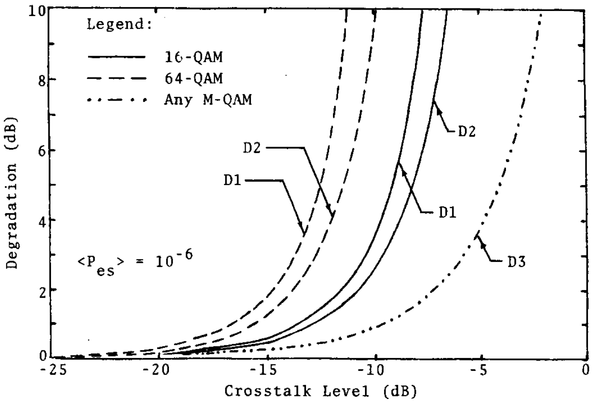
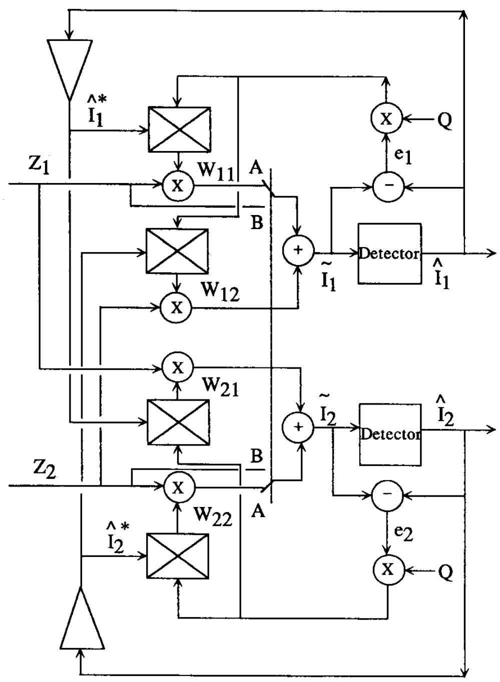


Fig. 3. Degradation for dual-channel M-QAM, $M = 16$, and $M = 64$.



Diagonalizer	Switch Position	Detector
D1	B	Regular QAM
D2	B	Figure 5
D3	A	Regular QAM

Fig. 4. Block diagram of the adaptive Di, $i = 1, 2, 3$ diagonalizer.

using an automatic gain control (AGC) device. However, this can also be accomplished by a modification of the detector of Fig. 4, switch in position B, as shown in Fig. 5. It is straightforward to show that S_{iR} and S_{iI}, $i = 1, 2$ in Fig. 5 converge to $(1 - r^2 \cos \phi)^{-1}$, as required.

We will do this for S_{1R}, and the reader can verify the others. We will also make the simplifying assumption that W_{12} has already converged to $-T_{12}$, and that S_{1R} has been fixed at unity during the convergence time of W_{12}. It must be stressed that this is not necessary for convergence of $D2$, but it does simplify the

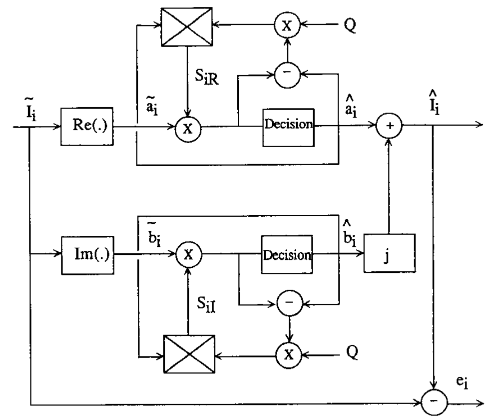


Fig. 5. Detector to be used for $D2$ diagonalizer.

analysis. As a matter of fact, the computer simulation mentioned below allowed W_{12} and S_{1R} to converge simultaneously.

From Fig. 5,

$$S_{1R}(k+1) = S_{1R}(k) + Qe_{1R}\hat{a}_1 \tag{9}$$

where $e_{1R} = \hat{a}_1 - S_{1R}\tilde{a}_1$; $\tilde{a}_1 = \text{Re}(Z_1 + W_{12}Z_2) = (1 - r^2 \cos \phi)a_1 + r^2 \sin \phi_1 b_1 + N_{1R} - r \cos \phi_1 N_{2R} + r \sin \phi_1 N_{2I}$ and Q is a constant chosen small enough to ensure convergence of the iteration process. Under steady state conditions, if we are to have $S_{1R}(k)$ approach the optimum value S_{1R0}, then we must have

$$E[e_{1R}\hat{a}_1] = 0 \tag{10}$$

where the expectation is with respect to a_1. Assuming $\hat{a}_1 = a_1$ most of the time, and solving (10) gives the desired result, i.e., $S_{1R0} = (1 - r^2 \cos \phi)^{-1}$.

All three implementations were simulated on a computer, and the weights converged to the analytical values. In the simulations, detected sequences $\hat{a}_i$ and $\hat{b}_i$ were replaced by the transmitted sequences a_i, b_i, $i = 1, 2$. Hence, the results apply directly to real systems which use a training sequence initially as discussed in [7], or to systems for which P_{es} is low enough to ensure $\hat{a}_i = a_i$, $\hat{b}_i = b_i$, $i = 1, 2$, most of the time, in spite of the crosstalk.

VI. Conclusion

In this letter, we have compared the probability of symbol error performances of three distinct diagonalizers, and have concluded that the $D3$ diagonalizer is the best and that for large signal-to-noise ratios, the MMSE canceller reduces to the $D3$ diagonalizer. Any comparison to other schemes of crosstalk cancellation should be based on $D3$'s performance.

Receiver structures were also discussed. It was shown that Kavehrad's structure given in [1] actually implements $D3$, and that $D1$ is implemented by constraining two weights of the $D3$ diagonalizer to be unity. A structure was also proposed for the $D2$ diagonalizer. It was also noted that the channel parameters can easily be recovered from the $D1$ and $D2$ diagonalizers.

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